<u>PreCalculus Summer Assignment</u>

Welcome to PreCalculus! We are excited for a fabulous year. Your summer assignment is available digitally on the Lyman website. **You are expected to print your own copy.**

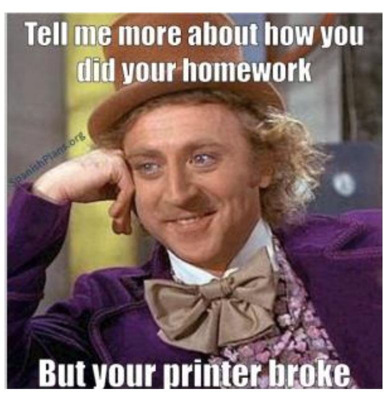
<u>Expectations:</u>

- These are topics you have covered in previous math courses, thus you are expected to complete <u>EVERY</u> problem!
- If you need extra help with any of the topics, you can use Khan Academy or other video tutorial websites, such as YouTube, etc. by searching by the topic name.
- We suggest making a completion plan as it is due the first day of school – **NO EXCEPTIONS!**
- **PLAN AHEAD** – You are to print the entire packet (including examples). You have the entire summer to locate a printer (such as a local library) so <u>no excuses</u>!

<u>Directions:</u>

- These questions are to be done **without** a calculator!
- You should read through the examples shown before each set of practice problems prior to trying them on your own.
- Do all your work for every problem printed in your packet. Box or circle your final answer. **NO WORK = NO CREDIT**!
- Once you've completed your packet, write ONLY YOUR FINAL ANSWERS on the separate document page, labeled Answer Sheet.
- The Answer Sheet and the packet containing your corresponding work will be graded for ACCURACY!!!

Let's make it a great year! ☺



Topic #1: Factoring

Examples: Review the following examples before completing the practice problems on the next page.

Common Factor: $x^3 + x^2 + x = x(x^2 + x + 1)$

Difference of Squares: $x^2 - y^2 = (x + y)(x - y)$ or $x^{2n} - y^{2n} = (x^n + y^n)(x^n - y^n)$

Perfect Squares: $x^2 + 2xy + y^2 = (x + y)^2$

Perfect Squares: $x^2 - 2xy + y^2 = (x - y)^2$

Sum of Cubes: $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ -Trinomial unfactorable

Difference of Cubes: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ -Trinomial unfactorable

Grouping: $xy + xb + ay + ab = x(y + b) + a(y + b) = (x + a)(y + b)$

The term "factoring" usually means that coefficients are rational numbers. For instance, $x^2 - 2$ could technically be factored as $(x + \sqrt{2})(x - \sqrt{2})$ but since $\sqrt{2}$ is not rational, we say that $x^2 - 2$ is not factorable.
It is important to know that $x^2 + y^2$ is unfactorable.

Completely factor the following expressions.

1. $4a^2 + 2a$

$2a(a + 2)$

2. $x^2 + 16x + 64$

8 x 8 = 64
8 + 8 = 16
$(x + 8)(x + 8) = (x + 8)^2$

3. $4x^2 - 64$

$4(x^2 - 16)$
$4(x + 4)(x - 4)$

4. $5x^4 - 5y^4$

$5(x^4 - y^4)$
$5(x^2 + y^2)(x^2 - y^2)$
$5(x^2 + y^2)(x + y)(x - y)$

5. $16x^2 - 8x + 1$

$16x^2 - 4x - 4x + 1$
$4x(4x - 1) - 1(4x - 1)$
$(4x - 1)^2$

6. $9a^4 - a^2b^2$

$a^2(9 - b^2)$
$a^2(3a + b)(3a - b)$

7. $2x^2 - 40x + 200$

$2(x^2 - 10x + 100)$
$2(x - 10)^2$

8. $x^3 - 8$

$(x - 2)(x^2 + 2x + 4)$

9. $8x^3 + 27y^3$

$(2x + 3y)(4x^2 - 6xy + 9y^2)$

10. $x^4 - 11x^2 - 80$

$x^4 - 16x^2 + 5x^2 - 80$
$x^2(x^2 - 16) + 5(x^2 - 16)$
$(x^2 - 16)(x^2 + 5)$
$(x + 4)(x - 4)(x^2 + 5)$

11. $x^4 - 10x^2 + 9$

$x^4 - 9x^2 - x^2 + 9$
$x^2(x^2 - 9) - 1(x^2 - 9)$
$(x^2 - 1)(x^2 - 9)$
$(x + 1)(x - 1)(x + 3)(x - 3)$

12. $36x^2 - 64$

$4(9x^2 - 16)$
$4(3x + 4)(3x - 4)$

13. $x^3 - x^2 + 3x - 3$

$x^2(x - 1) + 3(x - 1)$
$(x - 1)(x^2 + 3)$

14. $x^3 + 5x^2 - 4x - 20$

$x^2(x + 5) - 4(x + 5)$
$(x + 5)(x - 2)(x + 2)$

15. $9 - (x^2 + 2xy + y^2)$

$9 - (x + y)^2$
$(3 + x + y)(3 - x - y)$

<u>Practice:</u> Factor each expression completely.

1. $x^2 + 5x + 6$
2. $64m^2 - 25$
3. $2x^2 + 5x + 3$
4. $4p^2 - 9q^2$
5. $3x^2 + 4x + 1$
6. $9x^2y^2 - 49$
7. $3x^2 + 30x + 75$
8. $x^4 - 16$
9. $x^2 + 15x + 44$
10. $64 - y^2$
11. $2x^2 + 22x + 48$
12. $9x^4 - 4y^2$
13. $3x^2 - 20x - 7$
14. $a^2 + 8ab + 16b^2$
15. $x^3 - 11x^2 + 28x$
16. $25x^2 - 30xy + 9y^2$
17. $4x^2 - 12x + 9$
18. $9x^2y^2 - 6xy + 1$
19. $x^2 - 16$
20. $49x^2 + 1 + 14x$

Topic #2: Solving Quadratic Equations

Examples: Review the following examples before completing the practice problems on the next page.

Solve each equation by factoring.

1.

$$x^2 + 6x + 8 = 0$$

$$(x+4)(x+2) = 0$$

$$(x+4) = 0 \text{ or } (x+2) = 0$$

$$x = -4 \text{ or } x = -2$$

2.

$$2 = 6x^2 - x$$

$$0 = 6x^2 - x - 2$$

$$0 = (2x+1)(3x-2)$$

$$(2x+1) = 0 \quad \text{or} \quad (3x-2) = 0$$

$$2x = -1 \quad \text{or} \quad 3x = 2$$

$$x = -\tfrac{1}{2} \qquad x = \tfrac{2}{3}$$

3.

$$9x^2 - 6x + 1 = 0$$

$$(3x-1)(3x-1) = 0$$

$$(3x-1) = 0$$

$$3x = 1$$

$$x = \tfrac{1}{3}$$

Solve each equation by completing the square.

1. $x^2 + 6x + 8 = 0$

$$x^2 + 6x = -8$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

$$x^2 + 6x + 9 = -8 + 9$$

$$(x+3)^2 = 1$$

$$x + 3 = \pm 1$$

x = -4 or x = -2

2. $2 = 6x^2 - x$

$$\frac{1}{3} = x^2 - \frac{1}{6}x$$

$$\left(\frac{b}{2}\right)^2 = \left(-\frac{\frac{1}{6}}{2}\right)^2 = \left(-\frac{1}{12}\right)^2 = \frac{1}{144}$$

$$\frac{1}{3} + \frac{1}{144} = x^2 - \frac{1}{6}x + \frac{1}{144}$$

$$\frac{49}{144} = \left(x - \frac{1}{12}\right)^2$$

$$\pm\frac{7}{12} = x - \frac{1}{12}$$

$$x = -\frac{1}{2} \quad or \quad x = \frac{2}{3}$$

Solve by using the Quadratic Formula.

1. $2x^2 - 5x - 3 = 0$

Identify a, b, and c. Watch your signs carefully. $a = 2, b = -5, c = -3$

Write the Quadratic Formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute a, b, and c into the formula and do the initial calculations. $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$

$$x = \frac{5 \pm \sqrt{25 - (-24)}}{4}$$

Simplify the $\sqrt{\ }$. $x = \frac{5 \pm \sqrt{49}}{4}$

Calculate both values of x. $x = \frac{5+7}{4} = \frac{12}{4} = 3$ or $x = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2}$

The solutions are $x = 3$ or $x = -\frac{1}{2}$.

<u>Practice:</u> Solve each equation.

1. $3x^2 - 7x - 6 = 0$
2. $3x^2 + 10 = -11x$
3. $x^2 + 5x = -4$
4. $6x - 9 = x^2$
5. $6x^2 + 5x - 4 = 0$
6. $6x^2 = x + 15$

<u>Practice:</u> Solve each equation by using the Quadratic Formula.

1. $x^2 - x - 2 = 0$
2. $-2 - 2x^2 = 4x$
3. $6 - 4x + 3x^2 = 8$
4. $4x^2 + x - 1 = 0$
5. $x(-3x + 5) = 7x - 10$
6. $(5x + 5)(x - 5) = 7x$

Topic #3: Solving Advanced Equations

Examples: Review the following examples before completing the practice problems.

1. Solve $|2x - 3| = 7$

Separate into two cases.	$2x-3=7$	or	$2x-3=-7$
Add 3.	$2x=10$	or	$2x=-4$
Divide by 2.	$x=5$	or	$x=-2$

2. Solve $x(2x - 4) = (2x + 1)(x + 5)$

$$2x^2-4x=2x^2+11x+5$$

$$-4x=11x+5$$

$$-15x=5$$

$$x=\frac{5}{-15}=-\frac{1}{3}$$

Practice: Solve each equation. Find all solutions.

1. $\frac{3x+1}{2}=5$

2. $\sqrt{2x+5}=10$

3. $10-(x+7)=5$

4. $\sqrt{x}-3=7$

5. $(x+1)^2=81$

6. $\frac{x}{2}-\frac{x}{5}=3$

7. $|x-2|=5$

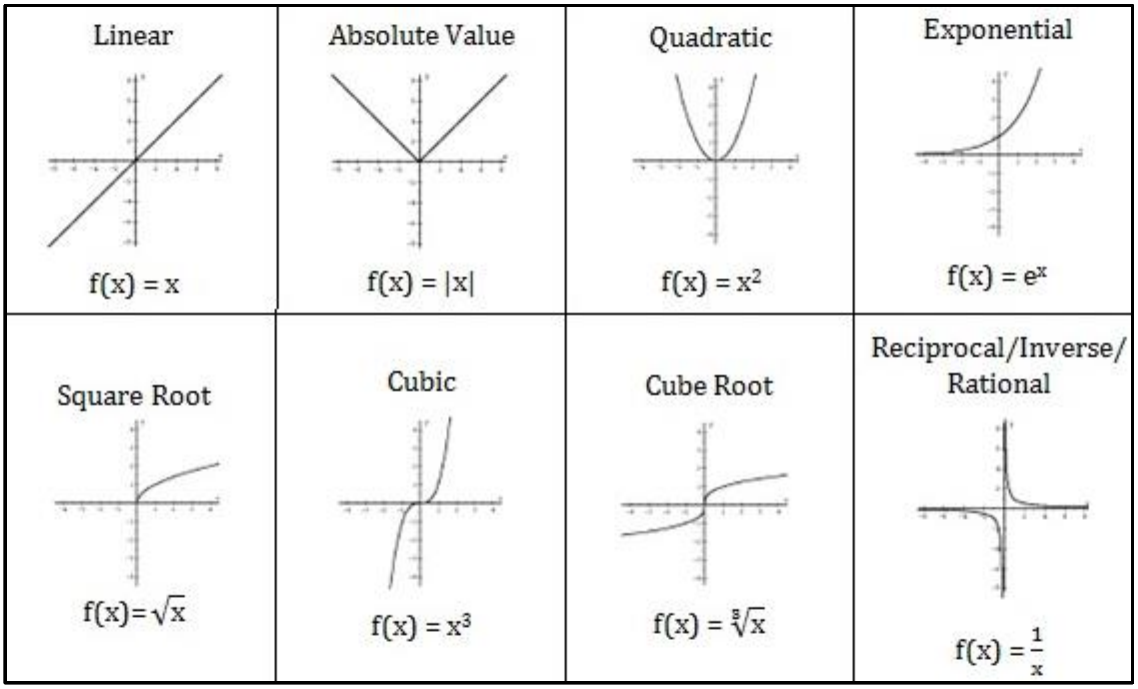
8. $2\sqrt{x-3}=8$

9. $x^2+5=4$

Topic #4: Transforming Parent Functions

Examples: Review the following examples before completing the practice problems on the next page.

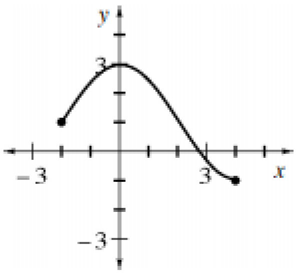
Common Parent Graph Functions



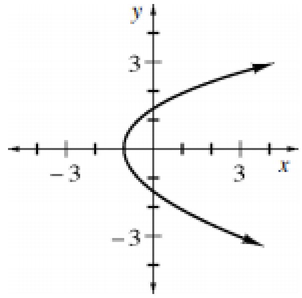
Transformation Rules		
Function Notation	**Type of Transformation**	**Change to Coordinate Point**
f(x) + d	Vertical translation up d units	$(x,y)\to(x,y+d)$
f(x) - d	Vertical translation down d units	$(x,y)\to(x,y-d)$
f(x + c)	Horizontal translation left c units	$(x,y)\to(x-c,y)$
f(x - c)	Horizontal translation right c units	$(x,y)\to(x+c,y)$
-f(x)	Reflection over x-axis	$(x,y)\to(x,-y)$
f(-x)	Reflection over y-axis	$(x,y)\to(-x,y)$
af(x)	Vertical stretch for $\lvert a\rvert > 0$	$(x,y)\to(x,ay)$
af(x)	Vertical compression for $0<\lvert a\rvert<1$	$(x,y)\to(x,ay)$
f(bx)	Horizontal compression for $\lvert b\rvert > 0$	$(x,y)\to\left(\frac{x}{b},y\right)$
f(bx)	Horizontal stretch for $0<\lvert b\rvert<1$	$(x,y)\to\left(\frac{x}{b},y\right)$

A relation in which each input has only one output is called a **function**.

$g(x)$



$f(x)$

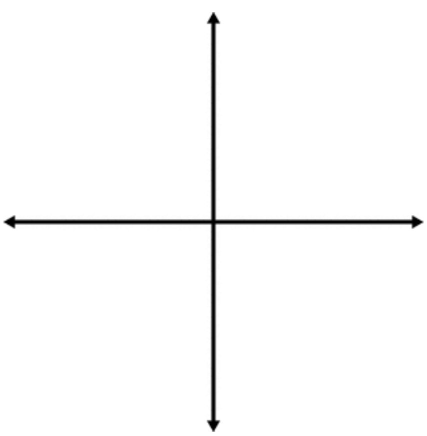


$g(x)$ is a function: each input (x) has only one output (y).
$g(-2)=1$, $g(0)=3$, $g(4)=-1$, and so on.

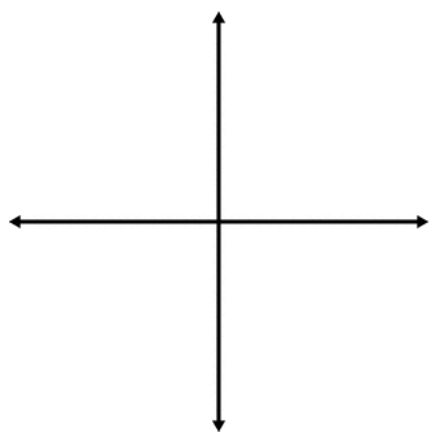
$f(x)$ is not a function: each input greater than -1 has two y-values associated with it.
$f(1)=2$ and $f(1)=-2$.

<u>**Practice:**</u> **For each of the following equations, state the parent equation and then sketch its graph. Be sure to include any key points (i.e., vertex, inflection point, etc.).**

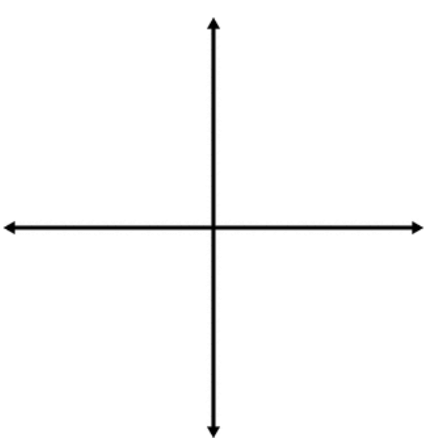
1. $y=-\frac{1}{3}(x+4)^2+7$



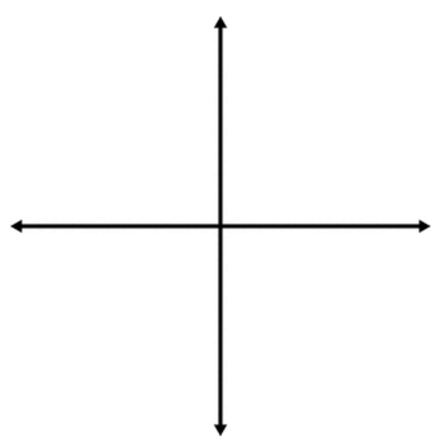
2. $y=|x+5|-2$



3. $y=\frac{1}{x+1}+10$

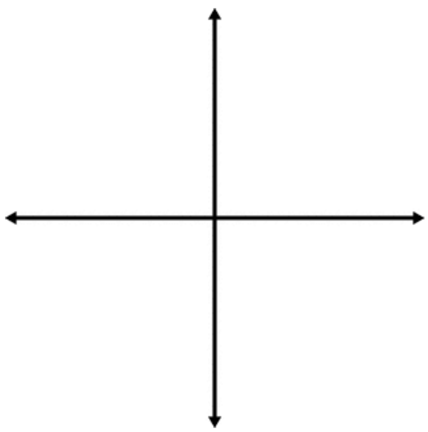


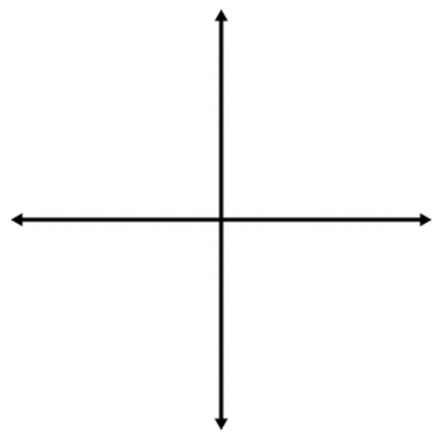
4. $y=2^x-8$



5. $y = -(x-2)^3 + 1$

6. $y = \sqrt{x+7}$

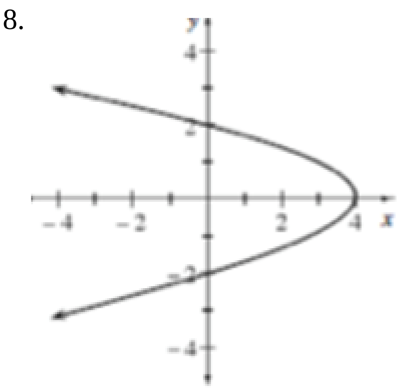




Practice: **For each of the following problems, state whether or not it is a function. Explain your answer.**

7.

8.



9. $y = 7 \pm \sqrt{9 - x^2}$

10. $y = 3(x-4)^2$

Topic #5: Simplifying Rational Expressions

Examples: Review the following examples before completing the practice problems.

Simplify each expression completely. Assume the denominator does not equal zero.

1.

$$\frac{12(x-1)^3(x+2)}{3(x-1)^2(x+2)^2}=\frac{4\cdot 3(x-1)^2(x-1)(x+2)}{3(x-1)^2(x+2)(x+2)} \qquad =\frac{4(x-1)}{(x+2)}$$

2.

$$\frac{x^2-6x+8}{x^2+4x-12}=\frac{(x-4)(x-2)}{(x+6)(x-2)} \qquad =\frac{(x-4)}{(x+6)}$$

Practice: Simplify each of the following expressions completely. Assume the denominator does not equal zero.

1. $\frac{2(x+3)(x-2)}{6(x-2)(x+2)}$

2. $\frac{15(x-1)(7-x)}{25(x+1)(x-7)}$

3. $\frac{36(y+4)(y-16)}{32(y+16)(16-y)}$

4. $\frac{(x+3)^2(x-2)^4}{(x+3)^4(x-2)^3}$

5. $\frac{12(x-7)(x+2)^4}{20(x-7)^2(x+2)^5}$

6. $\frac{x^2+5x+6}{x^2+x-6}$

7. $\frac{2x^2+x-3}{x^2+4x-5}$

8. $\frac{x^2+4x}{2x+8}$

9. $\frac{x^2-1}{(x+1)(x-2)}$

10. $\frac{x^2-4}{x^2+x-6}$

11. $\frac{x^2-16}{x^3+9x^2+20x}$

12. $\frac{2x^2-x-10}{2x^2+7x+2}$

Topic #6: Systems of Equations

Examples: Review the following examples before completing the practice problems on the next page.

Solve each system using substitution.

1. y = 2x
   x + y = 9

$$x+y=9$$

Replace y with $2x$, and solve.

$$x+(2x)=9$$
$$3x=9$$
$$x=3$$

$$y=2x$$

Since $x=3$,

$$y=2(3)$$
$$y=6$$

Answer: (3, 6) ← **this is the point on the graph where these two lines would intersect**

2. y = 7 – 3x
   3x + y = 10

$$3x+y=10$$
$$3x+(7-3x)=10$$
$$3x-3x+7=10$$
$$7\neq 10$$

Answer: no solution

Solve the system using elimination.

3. x + 3y = 7
   4x – 7y = -10

To use the Elimination Method, one of the terms in one of the equations needs to be opposite of the corresponding term in the other equation. One of the equations can be multiplied to make terms opposite. For example, in the system at right, there are no terms that are opposite. However, if the first equation is multiplied by –4, then the two equations will have $4x$ and $-4x$ as opposites. The first equation now looks like this: $-4(x+3y=7) \rightarrow -4x+(-12y)=-28$. When multiplying an equation, be sure to multiply all the terms on *both* sides of the equation. With the first equation rewritten, the system of equations now looks like this:

$$x+3y=7$$
$$4x-7y=-10$$

$$-4x+(-12y)=-28$$
$$4x-7y=-10$$

Since $4x-7y$ is equivalent to -10, they can be added to both sides of the first equation:

$$\begin{array}{rr} -4x+(-12y) & =-28 \\ +\quad 4x-7y & -10 \\ \hline -19y & =-38 \\ y & =2 \end{array}$$

Now any of the equations can be used to find x:

Since $4x-7y=-10$ and $y=2$,

$$4x-7(2)=-10$$
$$4x-14=-10$$
$$4x=4$$
$$x=1$$

The solution to the system of equations is $(1,2)$.

<u>Practice:</u> Solve each of the following systems. Be sure to state your answer as an ordered pair. Explain what the answer tells you about the graphs of the equations.

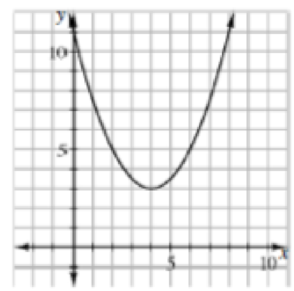
1. $2x - 3y = -19$
   $-5x + 2y = 20$

2. $8x + 2y = 18$
   $-6x + y = 14$

3. $12x - 16y = 24$
   $y = \frac{3}{4}x - \frac{3}{2}$

4. $\frac{1}{2}x - 7y = -15$
   $3x - 4y = 24$

<u>Practice:</u> The graph of y = 0.5(x – 4)^2 + 3 is shown at the right. Use the graph to solve each of the following equations. Explain how you got your answer.



5. $\frac{1}{2}(x-4)^2 + 3 = 3$

6. $\frac{1}{2}(x-4)^2 + 3 = 5$

7. $\frac{1}{2}(x-4)^2 + 3 = 1$

8. $\frac{1}{2}(x-4)^2 = 8$

<u>Practice:</u> Solve each of the following systems of equations algebraically. What does the solution tell you about the graph of the system?

9. $y = -3(x-4)^2 - 2$
   $y = -\frac{4}{7}x + 4$

10. $x + y = 0$
    $y = (x-4)^2 - 6$

11. Adult tickets for the *Mr. Moose's Fantasy Show on Ice* are \$6.50, while a child's ticket is only \$2.50. At Tuesday night's performance, 435 people were in attendance. The box office brought in \$1667.50 for that evening. How many of each type of ticket were sold?

Topic #7: Inverses

Examples: Review the following examples before completing the practice problems on the next page.

1. Find the inverse rule for each function below.

a. $f(x) = \frac{x-6}{3}$

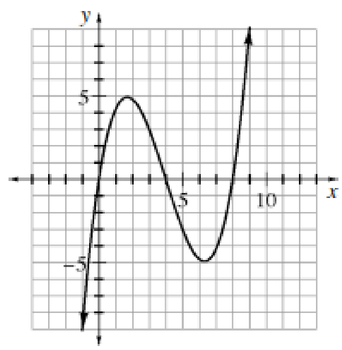
$$f(x) = \frac{x-6}{3}$$
$$y = \frac{x-6}{3}$$
$$x = \frac{y-6}{3}$$
$$3x = y - 6$$
$$3x + 6 = y$$

b. $g(x) = (x+4)^2 + 1$

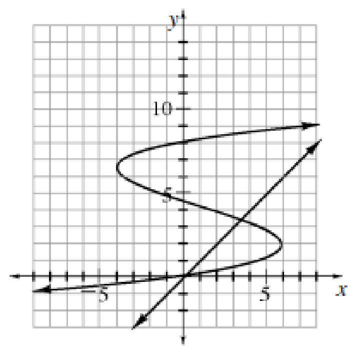
$$g(x) = (x+4)^2 + 1$$
$$y = (x+4)^2 + 1$$
$$x = (y+4)^2 + 1$$
$$x - 1 = (y+4)^2$$
$$\pm\sqrt{x-1} = y + 4$$
$$-4 \pm \sqrt{x-1} = y$$

2. The graph of f(x) = $0.2x^3 - 2.4x^2 + 6.4x$ is shown to the right. Graph the inverse of this function.

Following the algorithm for determining the equation for an inverse, as we did above, would be difficult here. The students do not have a method for solving cubic equations. Nevertheless, students can graph the inverse because they know a special property about the graphs of functions and their inverses: they are symmetrical about the line $y = x$.



If we add the line $y = x$ to the graph, the inverse is the reflection across this line. Here we can fold the paper along the line $y = x$, and trace the result to create the reflection.



3. Verify that $f(x) = \frac{2x-1}{7}$ and $g(x) = \frac{7x+1}{2}$ are inverse functions by composition.

$f(x) = \frac{2x-1}{7}$ $\qquad$ $g(x) = \frac{7x+1}{2}$

$$f\left(g(x)\right) = f\left(\frac{7x+1}{2}\right) = \frac{2\left(\frac{7x+1}{2}\right)-1}{7} = \frac{7x+1-1}{7} = \frac{7x}{7} = x$$

$$g\left(f(x)\right) = g\left(\frac{2x-1}{7}\right) = \frac{7\left(\frac{2x-1}{7}\right)+1}{2} = \frac{2x-1+1}{2} = \frac{2x}{2} = x$$

Since $f\left(g(x)\right) = g\left(f(x)\right) = x$, the functions are inverses.

Practice: **Find the inverse of each of the following functions.**

1. $y = -\frac{3}{4}x + 6$

2. $f(x) = x^2 + 6$

3. $f(x) = \frac{3}{x} + 6$

4. $g(x) = (x + 1)^2 - 3$

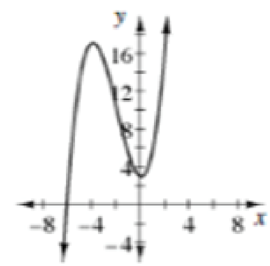
5. $y = 3 + \sqrt{x - 4}$

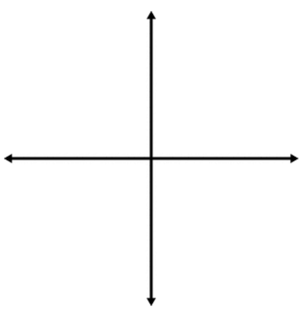
Practice: **Sketch the graph of the inverse of each of the following functions.**

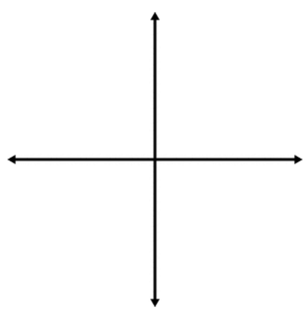
6. $y = \frac{x}{6} + 2$

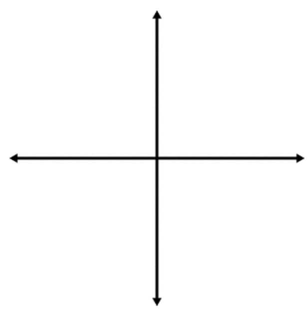
7. $f(x) = 2x^2 - 1$

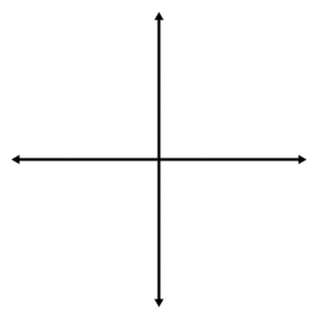
8. $y = \frac{1}{5x}$

9. 









Practice: **For each of the following pairs of functions, determine f(g(x)) and g(f(x)), then use the result to decide whether or not f(x) and g(x) are inverses of each other.**

10. $f(x) = x + 5$
    $g(x) = \frac{1}{x+5}$

11. $f(x) = \frac{2}{3x}$
    $g(x) = \frac{3x}{2}$

12. $f(x) = \frac{2}{3}x + 6$
    $g(x) = \frac{3(x-6)}{2}$

Topic #8: Trigonometry

Examples: Review the following examples before completing the practice problems on the next page.

1. Converting between degrees and radians:

Convert $\boxed{15°}$ to exact radian measure.

$$15° * \boxed{\frac{\pi\ rad}{180°}} = \frac{15\pi}{180}\ rad = \boxed{\frac{\pi}{12}\ rad}$$

Convert $\boxed{315°}$ to exact radian measure.

$$315° * \boxed{\frac{\pi\ rad}{180°}} = \frac{315\pi}{180}\ rad = \boxed{\frac{7\pi}{4}\ rad}$$

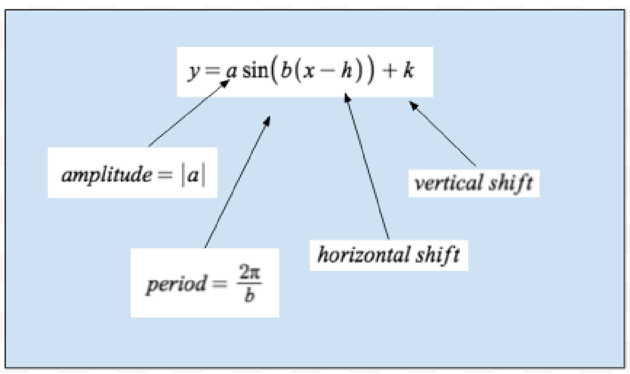
Convert $\boxed{\frac{\pi}{5}\ rad}$ to exact degree measure.

$$\left(\frac{\pi}{5}\ rad\right) * \boxed{\frac{180°}{\pi\ rad}} = \frac{180°}{5} = \boxed{36°}$$

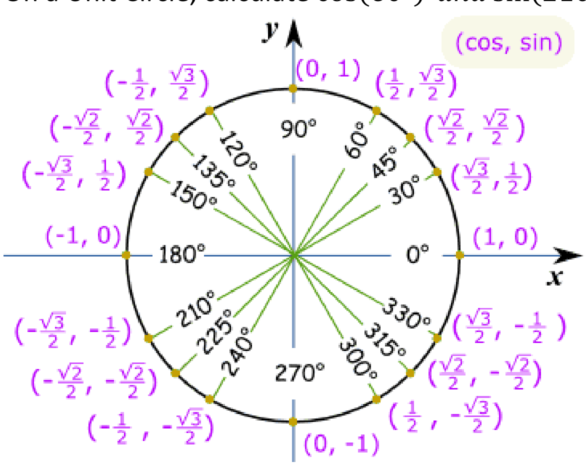
Convert $\boxed{\frac{3\pi}{8}\ rad}$ to exact degree measure.

$$\left(\frac{3\pi}{8}\ rad\right) * \boxed{\frac{180°}{\pi\ rad}} = \frac{540°}{8} = \frac{135°}{2} = \boxed{67.5°}$$

2. Graphing trigonometric functions in the form:



3. On a Unit Circle, calculate $\cos(60°)$ *and* $\sin(210°)$.



a) $\cos(60°) = \frac{1}{2}$

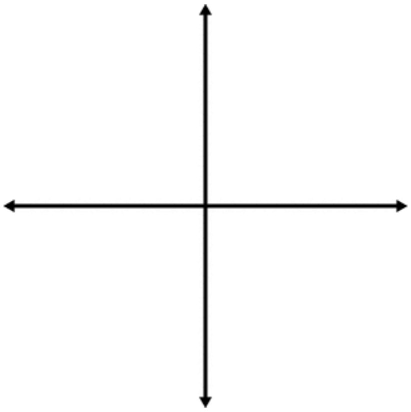
{since this is the x-value at 60°}

b) $\sin(210°) = -\frac{1}{2}$

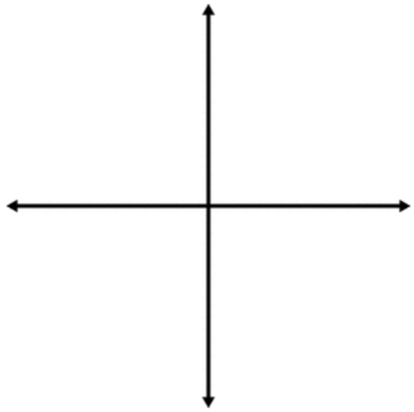
{since this is the y-value at 210°}

Practice: **Graph each of the following trigonometric equations.**

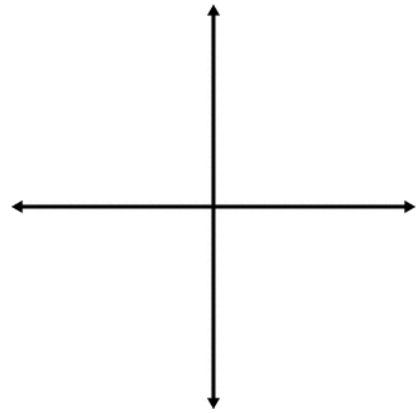
1. $y = \sin(x)$



2. $y = \cos(x)$



3. $y = \tan(x)$



Practice: **Find each of the following values without a calculator, but by using what you know about right triangle trigonometry, the unit circle, and special right triangles.**

4. $\cos(180°)$

5. $\sin(360°)$

6. $\tan(45°)$

7. $\cos(-90°)$

8. $\sin(150°)$

9. $\tan(240°)$

Practice: **Convert each of the following angle measures.**

10. $60°$ to radians

11. $170°$ to radians

12. $315°$ to radians

13. $-\frac{3\pi}{4}$ radians to degrees

Practice: **For each equation listed below, state the amplitude and period.**

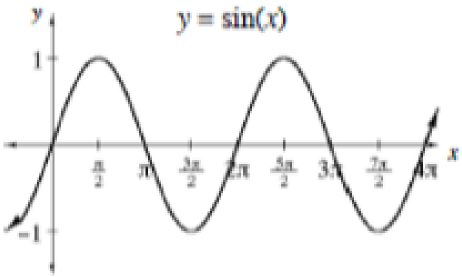
14. $y = 2\cos(3x) + 7$

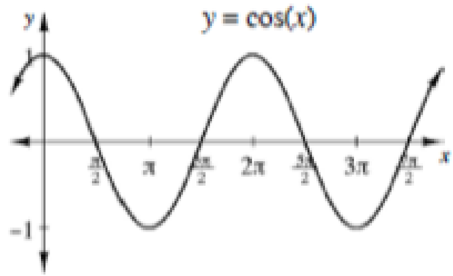
15. $y = \frac{1}{2}\sin(x) - 6$

16. $f(x) = -3\sin(4x)$

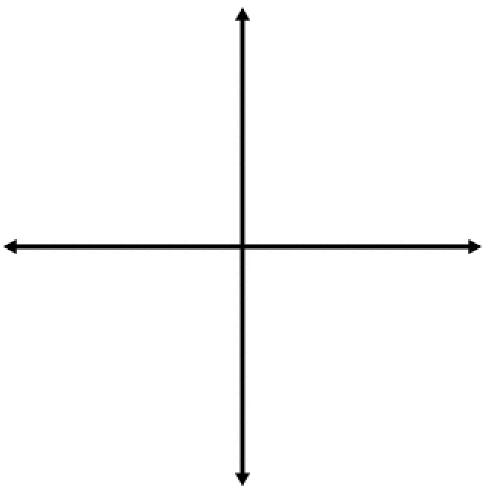
17. $y = \sin\left(\frac{1}{3}x\right) + 3.5$

<u>Practice:</u> Below are the graphs of sine and cosine. Use them to sketch the graphs of each of the following functions by hand. Use your graphing calculator to check your answer.

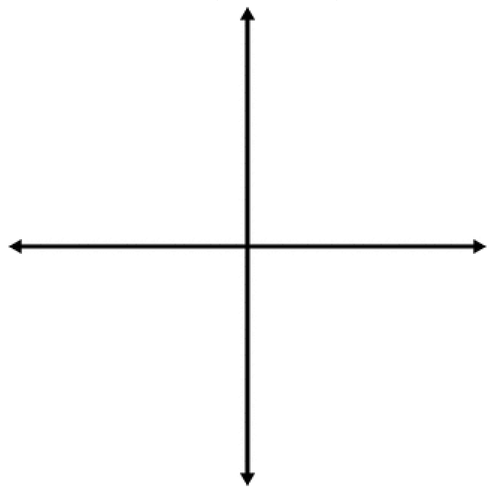




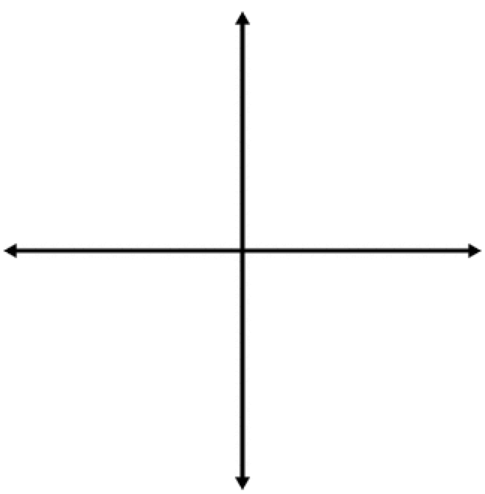
18. $y = -2\sin(x + \pi)$



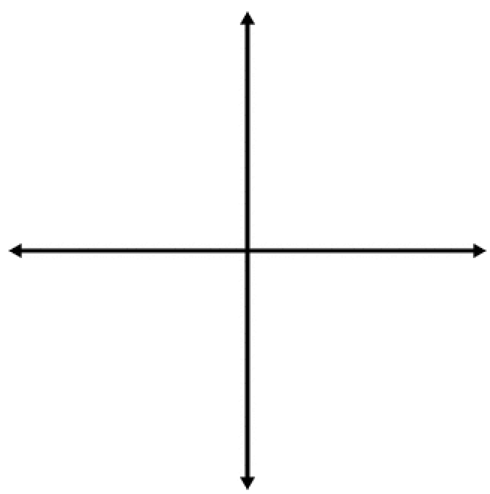
19. $f(x) = \cos\left(4\left(x - \frac{\pi}{4}\right)\right)$



20. $y = 3\cos\left(x + \frac{\pi}{4}\right) + 3$



21. $f(x) = 7\sin\left(\frac{1}{4}x\right) - 3$



Topic #9: Polynomials

Examples: Review the following examples before completing the practice problems on the next page.

1. Make a sketch of each of the following polynomials by using the orientation, roots, and degree.

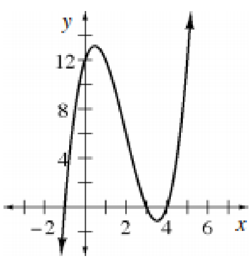
a. $f(x) = (x+1)(x-3)(x-4)$ b. $y = (x-6)^2(x+1)$

c. $p(x) = x(x+1)^2(x-4)^2$ d. $f(x) = -(x+1)^3(x-1)^2$

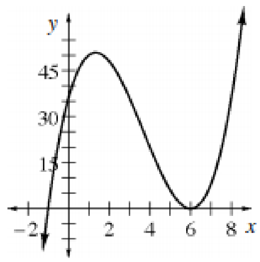
The roots of a polynomial are the x-intercepts, which are easily found in factored form. The values of x that would make each factor equal to 0 are the roots. In part a, the roots of the third degree polynomial are x = -1, 3, and 4. In part b, the roots of the third degree polynomial are 6 and -1. The degree tells you the maximum number of roots possible, and since this third degree polynomial has just two roots, you might ask where is the third root? x = 6 has what we call a multiplicity of 2, since this expression is squared and is thus equivalent to (x – 6)(x – 6). The graph will bounce off the x-axis at x = 6 and "bounce" off. The fifth degree polynomial in part c has three roots, 0, -1, and 4 with both -1 and 4 having a multiplicity of 2. The fifth degree polynomial in part d has two roots, -1 and 1, with 1 having a multiplicity of 2 and -1 having a multiplicity of 3. A multiplicity of 3 means that the graph will look cubic at the x-intercept (it will flatten out at the x-axis).

With the roots, we can sketch the graphs of each of these polynomials.

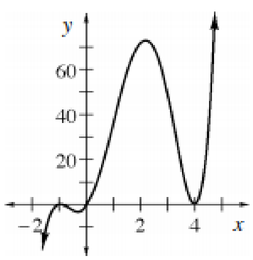
a.



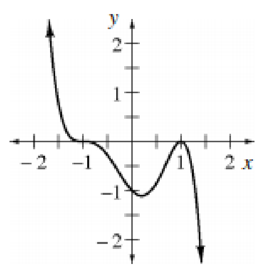
b.



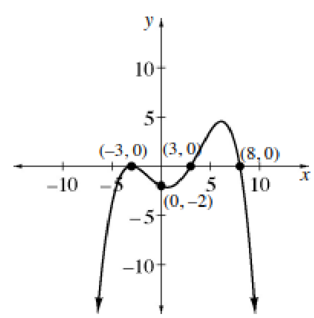
c.



d.



2. Write the exact equation of the graph shown at the right.
From the graph, we can see the x-intercepts are -3, 3, and 8 so the factors are (x + 3), (x – 3) and (x – 8). Since the graph bounces off the x-axis at -3, this factor has a multiplicity of 2 and should therefore be squared. So the function is $f(x) = a(x+3)^2(x-3)(x-8)$. We need to determine the value of a to have an exact equation.



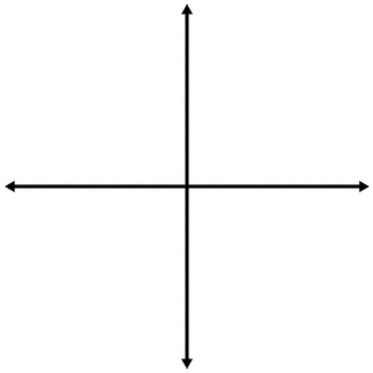
Using the fact that the graph passes through the point $(0, -2)$, we can write:

$$-2 = a(0+3)^2(0-3)(0-8)$$
$$-2 = a(9)(-3)(-8)$$
$$-2 = 216a$$
$$a = -\frac{2}{216} = -\frac{1}{108}$$

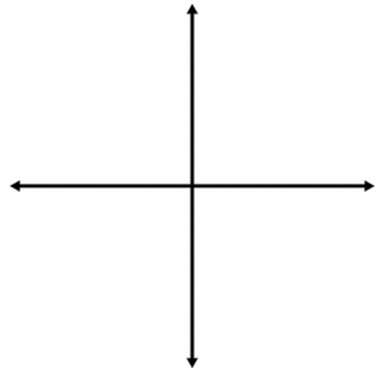
Therefore the exact equation is $f(x) = -\frac{1}{108}(x+3)^2(x-3)(x-8)$.

Practice: **Sketch the graph of each of the following polynomials.**

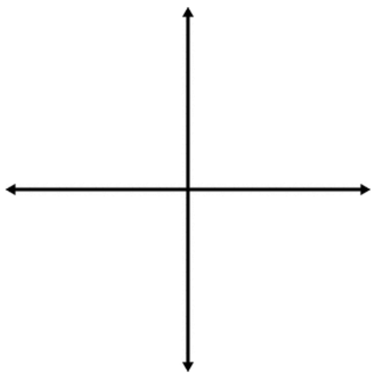
1. $y = (x+5)(x-1)^2(x-7)$



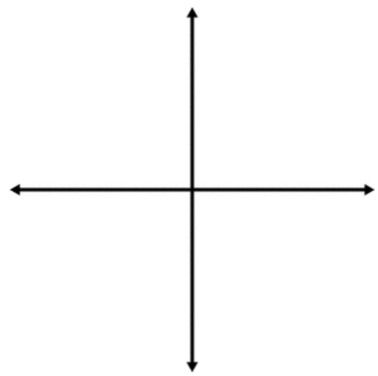
2. $y = -(x+3)(x^2+2)(x+5)^2$



3. $f(x) = -x(x+8)(x+1)$

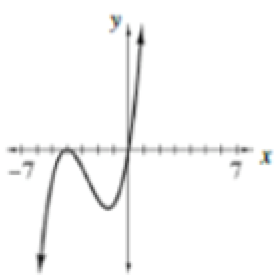


4. $y = x(x+4)(x^2-1)(x-4)$

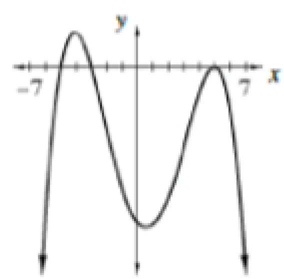


Practice: **Below are the complete graphs of some polynomial functions. Based on the shape and location of the graph, describe all the roots of the polynomial function, its degree, and orientation. Be sure to include information about multiplicities.**

5.

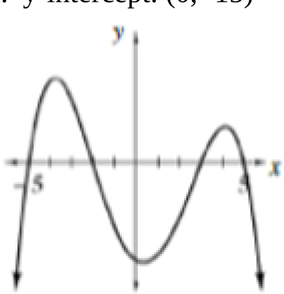


6.

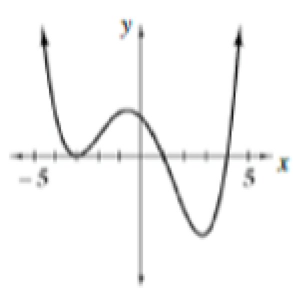


Practice: **Using the graphs below and given information, write the specific equation for each polynomial function.**

7. y-intercept: (0, -15)



8. y-intercept: (0, 3)



Topic #10: Complex Numbers

Examples: Review the following examples before completing the practice problems on the next page.

1. Simplify each of the following expressions.

a. $3+\sqrt{-16}$ b. $(3+4i)+(-2-6i)$

c. $(4i)(-5i)$ d. $(8-3i)(8+3i)$

Remember that $i=\sqrt{-1}$. Therefore, the expression in (a) can be written as $3+\sqrt{-16}=3+4\sqrt{-1}=3+4i$. This is the simplest form; we cannot combine real and imaginary parts of the complex number. But, as is the case in part (b), we can combine real parts with real parts, and imaginary parts with imaginary parts: $(3 + 4i) + (-2 - 6i) = 1 - 2i$. In part (c), we can use the commutative rule to rearrange this expression: $(4i)(-5i) = (4 \cdot -5)(i \cdot i) = -20i^2$. However, remember that $i=\sqrt{-1}$, so $i^2=(\sqrt{-1})^2=-1$. Therefore, $-20i^2 = -20(-1) = 20$. Finally in part (d), we will multiply using methods we have used previously for multiplying binomials. You can use the Distributive Property or generic rectangles to compute this product.

$$
\begin{aligned}
(8-3i)(8+3i) &= 8(8)+8(3i)-3i(8)-3i(3i)\\
&= 64+24i-24i+9\\
&= 73
\end{aligned}
$$

	8	$-3i$
8	64	$-24i$
$3i$	$24i$	9

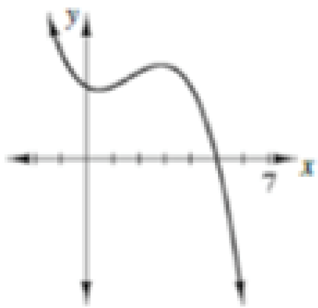
The two expressions in part (d) are similar. In fact they are the same except for the middle sign. These two expressions are called **complex conjugates**, and they are useful when working with complex numbers. As you can see, multiplying a complex number by its conjugate produces a real number! This will always happen. Also, whenever a function with real coefficients has a complex root, it always has the conjugate as a root as well.

Practice: Simplify the following expressions.

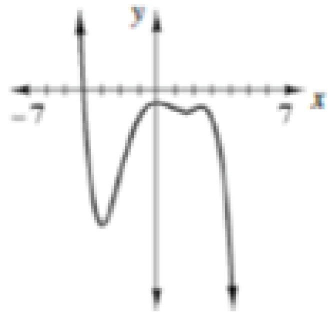
1. $(6+4i)-(2-i)$
2. $8i-\sqrt{-16}$
3. $(-3)(4i)(7i)$
4. $(5-7i)(-2+3i)$
5. $(3+2i)(3-2i)$
6. $(\sqrt{3}-5i)(\sqrt{3}+5i)$

<u>Practice:</u> Below are the complete graphs of some polynomial functions. Based on the shape and location of the graph, describe all the roots of the polynomial function. Be sure to include information such as whether roots are double or triple, real or complex, etc.

7.



8.



9. Write the specific equation for the polynomial function passing through the point (0, 5), and with roots $x = 5, x = -2$, and $x = 3i$.